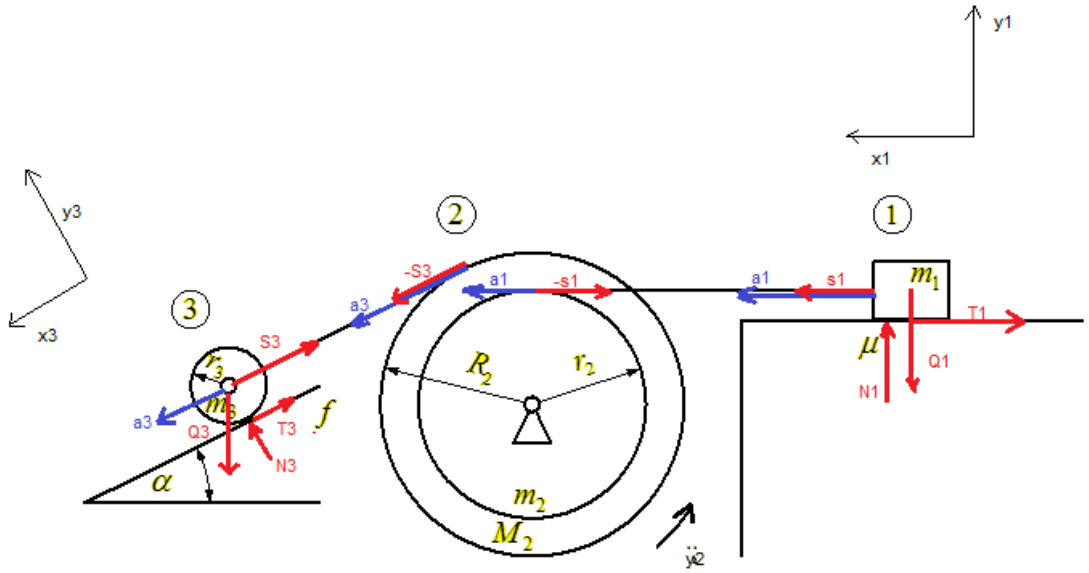

<u>Dane:</u>

Nr = 2

R_2 = 0,4 [m]

r_2 = 0,3 [m]

r_3 = 0,1 [m]

m_1 = 1 [kg]

M_2 = 4 [kg]

m_2 = 3 [kg]

m_3 = 1 [kg]

μ = 0,1

α = Nr + 30° = 32°

f = 0,00005 [m]

s_1 = 0,2 · Nr = 0,4 [m]

$\overrightarrow{S_\iota}$ – naprężenia

$\overrightarrow{Q_\iota}$ – siły grawitacji

$\overrightarrow{T_\iota}$ – siły tarcia (tarcie toczne z indeksem 3 i ślizgowe – 1)

$\overrightarrow{N_\iota}$ – siły reakcji podłoża

Dynamiczne równanie ruchu:

$$\begin{cases} m_1\ddot{x}_1 = S_1 - T_1 = S_1 - \mu Q_1 = S_1 - \mu m_1 g \\ m_1\ddot{y}_1 = -Q_1 + N_1 = 0 \end{cases}$$

$$I_2\ddot{\varphi}_2 = S_3R_2 - S_1r_2 \qquad I_2 = \frac{1}{2}[M_2R_2^2 + m_2r_2^2]$$

$$\begin{cases} m_3\ddot{x}_3 = m_3 g \cdot sin\alpha - S_3 - T_3 \\ m_3\ddot{y}_3 = N_3 - m_3 \cdot g \cdot cos\alpha \end{cases}$$

$$I_3\ddot{\varphi}_3 = T_3 r_3 - N_3 \cdot f \qquad I_3 = \frac{1}{2} m_3 r_3^2$$

Wzory na siłę tarcia:

- Toczne $T_3 = N_3 \frac{f}{r_3} = m_3 g\ cos\alpha \frac{f}{r_3}$
- Ślizgowe $T_1 = \mu N_1 = \mu m g$

Relacje:

$\ddot{x}_3 = r_3\ddot{\varphi}_3$ $\ddot{x}_1 = r_2\ddot{\varphi}_2$ $\dot{x}_3 = r_3\dot{\varphi}_3$ $\dot{x}_1 = r_2\dot{\varphi}_2$

$\ddot{x}_3 = R_2\ddot{\varphi}_2$ $\dot{x}_3 = R_2\dot{\varphi}_2$

$\dot{x}_i - prędkości\ liniowe\ (v)$ $\ddot{x}_i$– przyspieszenia liniowe (a)

$\dot{\varphi}_i - prędkości\ kątowe\ (\omega)$ $\ddot{\varphi}_i$– przyspieszenia kątowe (ε)

<u>Szukane:</u>

$\dot{x}_1, \dot{x}_2, \dot{x}_3, \ddot{x}_1, \ddot{x}_2, \ddot{x}_3, \dot{\varphi}_2, \dot{\varphi}_3, \ddot{\varphi}_2, \ddot{\varphi}_3$

$\dot{\varphi}_1 = 0$ $\ddot{\varphi}_2 = 0$ -> bryła, której nie obracamy

Bryła 1 porusza się ruchem jednostajnie przyspieszonym więc korzystamy ze wzorów:

$$s_1 = \frac{1}{2} a_1 t^2 \rightarrow t = \sqrt{\frac{2S_1}{a_1}}$$

$$v_1 = a_1 t = \sqrt{\frac{2S_1}{a_1}}, \qquad a_1 = \sqrt{2 s_1 a_1}$$

Jeżeli obliczymy $\ddot{x}_1$ to automatycznie znać będziemy pozostałe wyrazy, które wyliczamy ze wzorów:

$$\ddot{\varphi}_2 = \frac{\ddot{x}_1}{r_2}, \qquad \ddot{x}_3 = \frac{R_2}{r_2}\ddot{x}_1, \qquad \ddot{\varphi}_3 = \frac{R_2}{r_2 r_3}\ddot{x}_1, \qquad \ddot{x}_2 = \ddot{x}_3$$

$$\dot{x}_1 = \sqrt{2S_1\ddot{x}_1}, \qquad \dot{\varphi}_2 = \frac{1}{r_2}\sqrt{2S_1\ddot{x}_1}$$

$$\dot{x}_3 = \frac{R_2}{r_2}\sqrt{2S_1\ddot{x}_1}, \qquad \dot{\varphi}_3 = \frac{R_2}{r_2 r_3}\sqrt{2S_1\ddot{x}_1}, \qquad \dot{x}_2 = \dot{x}_3$$

$$S_1 = m_1\ddot{x}_1 + \mu m_1 g$$

$$S_3 = m_3 g\left(sin\alpha - cos\alpha\frac{f}{r_3}\right) - m_3\frac{R_2}{r_2}\ddot{x}_1 - I_3\frac{R_2}{r_2 r_3^2}\ddot{x}_1$$

$$\begin{cases} m_1\ddot{x}_1 = S_1 - \mu m_1 g \\ I_2\dfrac{\ddot{x}_1}{r_2} = S_3 R_2 - S_1 r_2 \\ m_3\dfrac{R_2}{r_2}\ddot{x}_1 = m_3 g\ sin\alpha - S_3 - \dfrac{1}{r_3}\left(I_3\dfrac{R_2}{r_2 r_3}\ddot{x}_1 + m_3 g\ cos\alpha\varphi\right) \end{cases}$$

$$S_1 = m_1\ddot{x}_1 + \mu m_1 g$$

$$S_3 = m_3 g sin\alpha - m_3\frac{R_2}{r_2}\ddot{x}_1 - I_3\frac{R_2}{r_2 r_3^2}\ddot{x}_1 - m_3 g\ cos\alpha\frac{\varphi}{r_3}$$

$$\frac{1}{r_2}I_2\ddot{x}_1 = m_3 g R_2\left(sin\alpha - cos\alpha\frac{\varphi}{r_3}\right) - m_3\frac{R_2^2}{r_2}\ddot{x}_1 - I_3\frac{R_2^2}{r_2 r_3^2}\ddot{x}_1 - m_1 r_2\ddot{x}_1 - \mu m_1 g r_2$$

$$\ddot{x}_1\left[\frac{I_2}{r_2} + m_3\frac{R_2^2}{r_2} + I_3\frac{R_2^2}{r_2 r_3^2} + m_1 r_2\right] = m_3 g R_2\left(sin\alpha - cos\alpha\frac{\varphi}{r_3}\right) - \mu m_1 g r_2$$

$$\ddot{x}_1 = \frac{m_3 g R_2 \left(sin\alpha - cos\alpha \frac{\varphi}{r_3}\right) - \mu m_1 g r_2}{\frac{I_2}{r_2} + m_3 \frac{R_2^2}{r_2} + I_3 \frac{R_2^2}{r_2 r_3^2} + m_1 r_2}$$

Obliczenia:

$$I_2 = \frac{1}{2}[M_2 R_2^2 + m_2 r_2^2]$$

$$I_2 = \frac{1}{2}[4 \cdot 0{,}4^2 + 3 \cdot 0{,}3^2]$$

$$\boldsymbol{I_2 = \frac{1}{2}[0{,}64 + 0{,}27] = 0{,}82}$$

$$I_3 = \frac{1}{2} m_3 r_3^2$$

$$\boldsymbol{I_3 = \frac{1}{2} \cdot 1 \cdot 0{,}1^2 = 0{,}005}$$

$$\ddot{x}_1 = \frac{m_3 g R_2 \left(sin\alpha - cos\alpha \frac{f}{r_3}\right) - \mu m_1 g r_2}{\frac{I_2}{r_2} + m_3 \frac{R_2^2}{r_2} + I_3 \frac{R_2^2}{r_2 r_3^2} + m_1 r_2}$$

$$\ddot{x}_1 = \frac{1 \cdot 10 \cdot 0{,}4 \left(sin32° - cos32° \frac{0{,}00005}{0{,}1}\right) - 0{,}1 \cdot 1 \cdot 10 \cdot 0{,}3}{\frac{0{,}82}{0{,}3} + 1 \cdot \frac{0{,}4^2}{0{,}3} + 0{,}005 \frac{0{,}4^2}{0{,}3 \cdot 0{,}1^2} + 1 \cdot 0{,}3} =$$

$$= \frac{2{,}103 - 0{,}3}{2{,}733 + 0{,}533 + 0{,}027 + 0{,}3} =$$

$$= \frac{1{,}803}{3{,}593} = \boldsymbol{0{,}474}$$

$$\ddot{\varphi}_2 = \frac{\ddot{x}_1}{r_2}$$

$$\ddot{\varphi}_2 = \frac{0{,}474}{0{,}3} = \mathbf{1{,}581}$$

$$\ddot{x}_3 = \frac{R_2}{r_2}\ddot{x}_1$$

$$\ddot{x}_3 = \frac{0{,}4}{0{,}3}0{,}474 = \mathbf{0{,}632}$$

$$\ddot{\varphi}_3 = \frac{R_2}{r_2 r_3}\ddot{x}_1$$

$$\ddot{\varphi}_3 = \frac{0{,}4}{0{,}3 \cdot 0{,}1}0{,}474 = \mathbf{6{,}323}$$

$$\ddot{x}_2 = \ddot{x}_3$$

$$\ddot{x}_2 = \mathbf{0{,}669}$$

$$\dot{x}_1 = \sqrt{2S_1\ddot{x}_1}$$

$$\dot{x}_1 = \sqrt{2 \cdot 0{,}4 \cdot 0{,}474} = \mathbf{0{,}616}$$

$$\dot{\varphi}_2 = \frac{1}{r_2}\sqrt{2S_1\ddot{x}_1}$$

$$\dot{\varphi}_2 = \frac{1}{0{,}3}\sqrt{2 \cdot 0{,}4 \cdot 0{,}474} = \mathbf{2{,}053}$$

$$\dot{x}_3 = \frac{R_2}{r_2}\sqrt{2S_1\ddot{x}_1}$$

$$\dot{x}_3 = \frac{0{,}4}{0{,}3}\sqrt{2 \cdot 0{,}4 \cdot 0{,}474} = \mathbf{0{,}821}$$

$$\dot{\varphi}_3 = \frac{R_2}{r_2 r_3}\sqrt{2S_1\ddot{x}_1}$$

$$\dot{\varphi}_3 = \frac{0{,}4}{0{,}3 \cdot 0{,}1}\sqrt{2 \cdot 0{,}4 \cdot 0{,}474} = \mathbf{8{,}213}$$

$$\dot{x}_2 = \dot{x}_3$$

$$\dot{x}_2 = \mathbf{0{,}845}$$

$$S_1 = m_1\ddot{x}_1 + \mu m_1 g$$

$$S_1 = 1 \cdot 0{,}474 + 0{,}1 \cdot 1 \cdot 10 = \mathbf{1,474}$$

$$S_3 = m_3 g sin\alpha - m_3 \frac{R_2}{r_2}\ddot{x}_1 - I_3 \frac{R_2}{r_2 r_3^2}\ddot{x}_1 - m_3 g\, cos\alpha \frac{\varphi}{r_3}$$

$$S_3 = 5{,}3 - 0{,}632 - 0{,}316 - 0{,}00424 = \mathbf{4,347}$$

Wyniki:

$I_2 = \mathbf{0,820}$

$I_3 = \mathbf{0,005}$

$\ddot{x}_1 = \mathbf{0,474}$

$\ddot{\varphi}_2 = \mathbf{1,581}$

$\ddot{x}_3 = \mathbf{0,632}$

$\ddot{\varphi}_3 = \mathbf{6,323}$

$\ddot{x}_2 = \mathbf{0,669}$

$\dot{x}_1 = \mathbf{0,615}$

$\dot{\varphi}_2 = \mathbf{2,053}$

$\dot{x}_3 = \mathbf{0,821}$

$\dot{\varphi}_3 = \mathbf{8,213}$

$\dot{x}_2 = \mathbf{0,845}$

$S_1 = \mathbf{1,474}$

$S_3 = \mathbf{4,347}$